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Stabilization of the Hierarchy in Brane-World Scenarios

Oriol Pujolàs[1]

We consider a class of warped brane models with topology $M_4 \times \Sigma \times S^1/Z_2$, where σ is a D_2-dimensional compact manifold, and two branes are placed at the orbifold fixed points. In a scenario where supersymmetry is broken not far below the cutoff scale, the hierarchy between the electroweak and the Planck scales is generated by a combination of the redshift and the large volume effects. We evaluate the effective potential induced by bulk scalar fields in these models and show that it can stabilize the moduli and the hierarchy without fine-tuning, provided that the internal space Σ is flat. We also comment on the relation between these models and the five-dimensional scalar-tensor models that describe them classically when the compactification scale is small.

KEY WORDS: extra dimensions; brane world; hierarchy problem; moduli stabilization.

1. INTRODUCTION

The possible resolution of the hierarchy problem in the context of the brane-world scenario (Antoniadis *et al.*, 1998; Arkani-Hamed *et al.*, 1998) has generated a great interest in the last years. Randall and Sundrum (RS) (1999) proposed a simple brane model in five dimensions where the 16 orders of magnitude separating the Planck and the electroweak scales are due the curvature of Anti-de Sitter (AdS) space. As in the old Kaluza–Klein theories, the size of the bulk is described by a four-dimensional field known as the *radion*. In models of both ADD and RS type, the hierarchy is determined by the radion vacuum expectation value (vev). Often, the radion is massless at tree level, as in the RS model (Garriga and Tanaka, 1999; Goldberger and Wise, 1999). Thus, in the brane-world scenario, the hierarchy problem is equivalent to the problem of *stabilizing* the radion at a suitable vev and with a large enough mass (Antoniadis *et al.*, 1998; Arkani-Hamed, 1998, 1999, 2001).

In the RS model, Goldberger and Wise (1999) proposed a mechanism to stabilize the radion by introducing a classical bulk field with appropriate boundary

[1] Yukawa Institute for Theoretical Physics, Kyoto University, Kitashirakawa-Oiwake-Cho Sakyo-ku, Kyoto 606-8502, Japan; e-mail: pujolas@yukawa.kyoto-u.ac.jp.

conditions. In the context of KK theories, Candelas and Weinberg and (1984) showed that the Casimir effect can stabilize the size of the extra dimensions. The possibility that quantum effects stabilize a large hierarchy in the RS model was considered in the literature (Flachi *et al.*, 2001a,b; Flachi and Toms, 2001; Garriga *et al.*, 2001; Goldberger and Rothstein, 2000; Toms, 2000) (see also Elizalde *et al.*, 2002; Himemoto and Sasaki, 2001; Kobayashi *et al.*, 2001; Moss *et al.*, 2003a,b; Mukohyama, 2001; Naylor and Sasaki, 2002; Najiri *et al.*, 2000; Sago *et al.*, 2001, for discussions of the possible relevance of quantum effects in cosmological brane-world scenarios and Brevik *et al.*, 2001, for finite temperature effects). The outcome was that generic bulk fields may stabilize the interbrane distance, but a large hierarchy is not naturally obtained, i.e., fine-tuning of the parameters is needed (and thus the hierarchy problem would be replaced by another fine tuning problem). However, the Casimir force due to a bulk gauge field naturally stabilizes the hierarchy (Garriga and Pomarol, 2003), generating a sizable radion mass.

The quantum effects in brane-world scenarios can be qualitatively different in more general models, providing new ways of stabilizing the radion which do not necessarily rely on the peculiar behavior of bulk gauge fields. Generically, we expect that the behaviour of the effective potential for the "moduli" should be qualitatively different once we go beyond the RS scenario. This is indicated (even in five dimensions) by the models with a bulk scalar field discussed (Garriga *et al.*, 2003).

In the RS model both the branes and the bulk space-time are maximally symmetric and thus any possible counterterm amounts to a renormalization of the brane tensions. However, this is not true in general. An explicit example is given in Garriga *et al.* (2003), where a class of 5D models with power-law warp factors is considered. These models have a global symmetry which is responsible for the masslessness of the moduli at tree level. However, this symmetry is anomalous, hence the 1-loop effective potential contains terms which do not scale appropriately and which therefore stabilize the moduli. Some of the 5-dimensional models considered in Garriga *et al.* (2003) can be obtained by dimensional reduction of $5 + D_2$-dimensional models. In this paper we shall focus on a class of higher dimensional models which includes those, and a detailed discussion of the relationship between these models is given in Section 5.

We shall consider spaces with line element given by

$$ds^2_{(D)} = e^{2\sigma(y)}\eta_{\mu\nu}\, dx^\mu\, dx^\nu + e^{2\rho(y)}R^2\gamma_{ij}\, dX^i\, dX^j + dy^2, \qquad (1)$$

where the coordinates x^μ parametrize four-dimensional Minkowski space M_4 and the coordinates χ^i cover a D_2-dimensional compact internal manifold σ. We locate two $D - 1 \equiv (4 + D_2)$-dimensional branes at the fixed points of the orbifolded dimension labeled by y. Such a metric is found as a solution of a D-dimensional theory of gravity plus certain "matter" fields. Depending on the field content, different warp factors σ and ρ can arise. Several situations have been studied in the literature (Davoudiasl *et al.*, 2002; Flachi and Pujolas, 2003; Gherghetta *et al.*, 2000; Gherghetta and Shaposhnikov, 2000; Gregory, 2000; Oda, 2001; Randjbar-Daemi and Shapashinikov, 2000a, b). Here, we focus on the case when both warp factors are exponential, $\sigma(y) = \rho(y) = -ky$. This corresponds to a generalization of the RS model in which the branes have more than four dimensions (with topology $M_4 \times \Sigma$) but still are of codimension one. As we will see, in such a case, both the redshift effect (Randall and Sundium, 1999) and the large volume effect (Antoniadis *et al.*, 1998; Arkani-Hamed *et al.* 1998) play a role in this case, and a large hierarchy is potentially easy to obtain. After describing the properties of this background, we discuss how the the Casimir forces can stabilize the brane positions and generate a large hierarchy without fine-tuning the parameters of the model.

This report is based on an article done in collaboration with Flachi *et al.* (2003). The plan of this paper is as follows. In Section 2 we present the model under consideration. In Section 3 we describe how the hierarchy arises in this model. In Section 4 we present the result for the effective potential induced by bulk fields propagating in this model and discuss the stabilization of the moduli. Finally we analyze the connection of this higher dimensional scenario to the five-dimensional model considered in Garriga *et al.* (2003) in Section 5, and we summarize the conclusions in Section 6.

2. MODEL

We consider two branes of codimension one with the topology $M_4 \times \Sigma$ sitting at the fixed points of a S^1/Z_2 orbifold with metric (1). Here, the manifold Σ is taken to be Einstein and compact. Specifically, we focus on the case when the two warp factors in (1) are equal and exponential, i.e., $\sigma(y) = \rho(y) = -k|y|$. The field content in the bulk consists of a G-invariant nonlinear sigma model parametrized by a set of bulk scalar fields ϕ^a together with a standard bulk gravity sector. This is described by the action

$$S = \int d^D x \sqrt{g(D)} \{ M^{D-2} \mathcal{R}_{(D)} - \Lambda - \partial_M \phi^{a\dagger} \partial^M \phi^a - \lambda(\phi^{a\dagger}\phi^a - \upsilon^2 \}$$
$$- \int d^{D-1} x \sqrt{g(D-1)+} \tau + - \int d^{D-1} x \sqrt{g(D-1)-} \tau - . \qquad (2)$$

In our notation, the higher dimensional bulk indices are $M, N \ldots$ and run over μ, i, y; the $(4 + D_2) \equiv D - 1$-dimensional brane indices are $A, B, \ldots$ and run over μ, i; $g^{(D)}_{MN}$ is the bulk metric and $g^{(D-1)\pm}_{AB}$ are the induced metrics on the branes. Finally, $\tau_\pm$ are the brane tensions, and M is the higher dimensional fundamental Planck mass.

The equations of motion of the sigma model sector can be written in the form (Flachi *et al.*, 2003)

$$\Box\phi^q = -\left(\frac{\partial_M\phi^b\partial^M\phi^{b\dagger}}{\upsilon^2}\right)\phi^a. \tag{3}$$

This equation allows hedgehog solutions for ϕ^a for suitable choices of the group G. Moreover, they have a constant profile along the orbifold and satisfy $\triangle_\gamma\phi^a = -L^2\phi^a$, where L is a "winding number," and $\triangle_\gamma$ is the Laplacian obtained from γ_{ij}.

The Einstein equations for this model when the sigma model scalars take such a hedgehog configuration have been studied in the literature (Gherghetta *et al.*, 2000; Gherghetta and Shaposhnikov, 2000; Oda, 2001). Solutions of the type (1) with $\sigma(y) = \rho(y) = -\kappa y$ exist as long as

$$\kappa = \sqrt{-4M^{2-D}\Lambda/(D-1)(D-2)}, \tag{4}$$

and $\Lambda < 0$. To obtain the space-time described previously, we take two copies of a slice of this D-dimensional space comprised between y_+ and y_-, corresponding to the brane locations. The two copies are glued together there.

In order for this to be a solution of our model (2), the brane tensions have to satisfy

$$\tau_\pm = \pm 4\sqrt{-(D-2)M^{D-2}\Lambda/(D-1)} = \pm 4(D-2)M^{(D-2)}k, \tag{5}$$

as a result of the junction conditions at the branes. Besides (4), the Einstein equations in the bulk relate the hedgehog parameters to the curvature of the internal manifold Σ as

$$\upsilon^2 = \frac{2D_2C}{L^2}M^{D-2}. \tag{6}$$

Here, the dimensionless constant C is defined through $\mathcal{R}^{(\gamma)}_{ij} = C\gamma_{ij}$, and $\mathcal{R}^{(\gamma)}_{ij}$ is the Ricci tensor computed out of γ_{ij}.

Associated to the sigma model scalars a number of Nambu Goldstone modes will be present. However, we shall assume that these couple to matter only through gravity so that their effects are negligible.

2.1. Moduli

We note that the parameter R, describing the volume of Σ, does not appear in the equations of motion even in the case of a curved internal space. The same holds for the positions of both branes, $y_\pm$. They correspond to flat directions in the action at the classical level. Since they correspond to light degrees of freedom, they are promoted to four-dimensional scalar fields.

In contrast with the RS model, these solutions are not homogeneous along the orbifold, even in the case when Σ is a torus. This is due to the compactness of Σ. In contrast with the RS model, here the positions of both branes are physically meaningful. However, it is clear that a scaling of R is equivalent to a shift in the positions of the branes $y_\pm$. Therefore, they are not independent. Rather, only two moduli are needed. In this paper we use the following combinations of the moduli: $a_\pm \equiv e^{-ky^\pm}$, the physical radii of Σ at the branes $R_\pm = a_\pm R$, the corresponding dimensionless values $r_\pm = a_\pm kR$, and $a \equiv e^{-\kappa(y_- - y_+)} = a\_/a_+$.

In addition to these moduli, the massless sector also contains the graviton zero mode. To take it into account, we can consider a general perturbation around the background of the form (1) as follows,

$$ds^2 = dy^2 + e^{2\sigma(y)}[\tilde{g}_{\mu\nu}(x)\,dx^\mu\,dx^\nu + R^2\gamma_{ij}\,dX^i\,dX^j], \tag{7}$$

where $\tilde{g}_{\mu\nu}(x)$ is a four-dimensional metric "close" to flat space. Substituting this metric back into the action (2), we obtain the kinetic term for $\tilde{g}$ coming from the bulk part (see Garriga *et al.*, 2003). The kinetic terms for the moduli $y_\pm$ come from the boundary terms. A computation analogous to that in Garriga *et al.* (2003) gives

$$S_{(4)} = -m_p^2 \int d^4x\sqrt{-\hat{g}}\left\{\hat{\mathcal{R}} + 2\frac{D_2}{D_2+2}(\hat{\partial}\ln\psi)^2 + 4\frac{D_2+3}{D_2+2}(\hat{\partial}\varphi)^2\right\}, \tag{8}$$

where the variables ψ and φ (Garriga *et al.*, 2003; Khaury *et al.*, 2001) are given by

$$a_+^{(D-3)/2} = \psi\,\cosh\varphi \quad \text{and} \quad a_-^{(D-3)/2} = \psi\,\sinh\varphi,$$

the (4D Enstein frame) is given by the metric $\hat{g}_{\mu\nu} = \psi^2\hat{g}_{\mu\nu}$, and the effective four-dimensional Planck mass is

$$m_p^2 = \frac{2}{D-3}\upsilon_\Sigma R^{D_2}M^{D-2}/k \quad \text{with} \quad \upsilon_\Sigma = \int_\Sigma \sqrt{\gamma}d^{D_2}X. \tag{9}$$

We note that the modulus ψ decouples in the limit $D_2 \to 0$, as expected, since this case corresponds to the usual RS model, where only one modulus is present.

We shall assume that the ((D − 1)-dimensional) matter fields $\chi_\pm^{(D-1)}$ are localized on each brane and so they couple universally to the corresponding induced metrics $g_{AB}^{(D-1)\pm}$ (recall A, B,... $= \mu, i$)

$$\begin{aligned} S^{\text{matt}} &= \sum_\pm \int d^{D-1}x\sqrt{-g^{(D-1)\pm}}\mathcal{L}^\pm\big(\chi^\pm_{(D-1)}, g_{AB}^{(D-1)\pm}\big) \\ &= \int d^4x \sum_\pm \sqrt{-g_\pm}a_\pm^{D_2}\mathcal{L}^\pm(\chi^\pm, g^\pm_{\mu\nu}). \end{aligned} \tag{10}$$

Defining the canonical fields

$$\hat{\psi} = 2\sqrt{\frac{D_2}{D_2+2}}m_p\delta \ln\psi, \quad \text{and} \quad \hat{\varphi} = 2\sqrt{2\frac{D_2+3}{D_2+2}}m_p\delta\varphi,$$

we obtain the equations of motion for the moduli

$$\hat{\Box}\hat{\psi} = \frac{1}{2}\sqrt{\frac{D_2}{D_2+2}}\frac{1}{mp}[\hat{T}_+ - 2\hat{\mathcal{L}}_+ + \hat{T}_- - 2\hat{\mathcal{L}}_-] \tag{11}$$

$$\hat{\Box}\hat{\psi} = -\frac{1}{\sqrt{2(D_2+3)(D_2+2)}m_p}\big[a^{(D_2+2)/2}(\hat{T}_+ + D_2\hat{\mathcal{L}}_+) + a^{-(D_2+2)/2}(\hat{T}_- + D_2\hat{\mathcal{L}}_-)\big].$$

As we explain in Section 3, we are interested in the case of a $\ll 1$ to have a substantial redshift effect arising from the warp factors. Unless otherwise stated, we shall set $\langle a_+\rangle = 1$, so that, with a good accuracy, $a_- \simeq a$, $\psi \simeq \psi_+ \simeq 1$, and $\varphi \simeq \psi_- \ll 1$.

Thus, from (11) we can read off the couplings to the two types of matter: $\hat{\psi}$ couples to the matter at either brane $\chi_\pm$, with a strength $\sim 1/m_p$. As for φ, the coupling to χ_- is quite large, of order $a^{-(D_2+2)/2}/m_p$, and to χ^+ is even smaller than Planckian, $\sim a^{(D_2+2)/2}/m_p$.

3. COMBINING ADD AND RS

In this section we set up a scenario where supersymmetry is broken at a scale η_{SUSY} not far below the cutoff scale M. This makes that a combination of both redshift and large volume effects generates quite easily the large hierarchy between the electroweak and the Planck scales.

From Eqs. (8) and (9), we see that the relation between the four-dimensional effective Planck mass and the higher dimensional one (in the four-dimensional effective theory using the Einstein frame metric $\hat{g}_{\mu\nu}$) is

$$m_p^2 \approx (MR)^{D_2}\frac{M}{k}M^2. \tag{12}$$

We shall assume that the masses of particles (located at $y = y_-$) are somewhat below the cutoff M. In the four-dimensional theory, these masses are redshifted down to $\sim aM$. Then, the EW/Planck hierarchy is given by

$$h^2 \equiv a^2\frac{M^2}{m^2p} \sim \frac{a^2}{(RM)^{D_2}}\frac{k}{M} \sim 10^{-32}. \tag{13}$$

Thus, the EW/Planck hierarchy h is explained in this model due to a combination of *redshift* (Randall and Sundrum, 1999) and *large volume* (Antoniadis *et al.*, 1998; Arkani-Hamed *et al.*, 1998) effects (even though the branes are of codimension

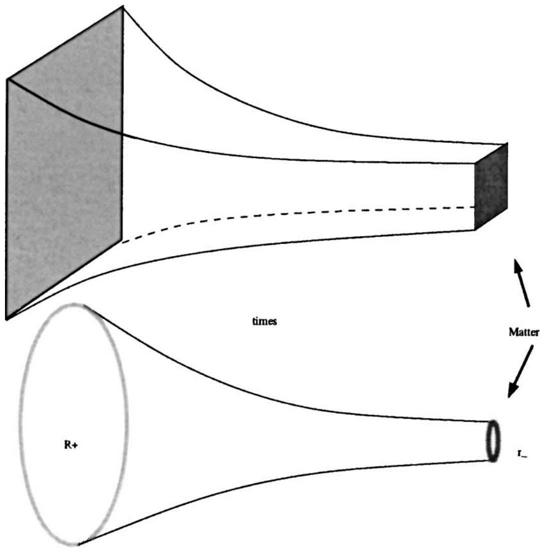

Fig. 1. Matter can propagate along the additional extra space $\sum$ of size $R_{\_}$, but gravity samples a much bigger space.

1). The crucial ingredient in order for the large volume effect to be efficient (aside from having a long orbifold) is that the additional extra space Σ exponentially grows as one moves away from the negative tension brane (see Fig. 1). In this way, matter is allowed to propagate along a small Σ, of size $R_{\_}$, whereas gravity is diluted since it propagates through a much larger Σ, of effective size R_+. Since the gauge interaction must not be diluted by an analogous effect, we have to assume that the compactification scale on the negative tension brane $1/R_{\_}$ is close to the fundamental cutoff M.

Our model solves the hierarchy problem in a fashion very similar to the models considered in Chacko *et al.* (2002) and Chacko and Nelson (2000), with two concentric branes embedded in a noncompact bulk (see also Burgess *et al.*, 2002; Multamaki and Vilja, 2002). In Chacko *et al.* (2002) and Chacko and Nelson (2000), the hierarchy is stabilized by a generalization of the Goldberger and Wise mechanism (Goldberger and Wise, 1999).

The main constraint that we have on the moduli comes from the result for the potential that we obtain in the next section (16). This is organized as a power series in $r_\pm = kR_\pm$, and can be trusted only when $1/R_+$ is larger than the curvature scale. The same holds for $1/R_{\_}$, since this is a factor a^{-1} larger [2] (recall that

[2] Incidentally, this corresponds to the physical situation where the size of the internal manifold Σ is *everywhere* smaller than the interbrane distance $\sim$ 1/k. This means that in a certain range of energy the model is effectively 5 dimensional. In Section 5, we derive the form of the dimensionally reduced theory down to 5 dimensions.

$R/R_+ = a$). So, we must assume a separation between the fundamental cutoff M and the curvature scale k at least of order a. This leads to the following scenario.

Consider a supersymmetric theory with SUSY broken at a scale ηSUSY. The bulk cosmological constant is expected to be proportional to η_{SUSY}^D. From Einstein equations, it is also given by $\Lambda \sim k^2 M^{D-2}$, which leads to

$$k \sim \left(\frac{\eta_{\mathrm{SUSY}}}{M}\right)^{D/2} M \ll M. \tag{14}$$

The important point is that even if SUSY is broken not far below the cutoff scale, this leads to a curvature scale k many orders of magnitude below M, due to the large exponent in (14). If the moduli $R_\pm$ are stabilized near the values $R_+ \sim 1/k$ and $R_- \sim 1/M$, then $a \sim k/M$ and from (13), the hierarchy is given by

$$h \sim \left(\frac{k}{M}\right)^{(D-2)/2} \sim \left(\frac{\eta_{\mathrm{SUSY}}}{M}\right)^{(D-1)/4}. \tag{15}$$

Note that the required hierarchy is obtained with η_{SUSY} within one order of magnitude of the cut off M for $D = 11$, and fewer than three orders of magnitude below M for $D = 6$.

This shows how the problem of the stabilization of a large hierarchy works in this model. Having introduced a small separation between the SUSY breaking and the cutoff scales, we obtain a stable very flat warped space-time, $k \ll M$. If some mechanism can stabilize the moduli $R_\pm$ near the values, $R_+ \sim 1/k$ and $R_- \sim 1/M$, then the effective Planck mass is very large as compared to the EW scale. Whether or not the effective potential (16) can do this job is addressed in the next section.

Let us discuss now for what range of the parameters the model is most interesting. As illustrated in Fig. 1, the branes are of codimension 1, so that matter can propagate through a physical extra dimensional space of size $\sim R_-$. The mass scales on this brane are redshifted by a factor a; thus, the mass of the first KK excitations of matter fields is $1/R$. Then, from collider physics, we have to set the compactification scale $1/R \gtrsim$ TeV, at least.

In contrast, gravity propagates through the whole bulk space. There are three kinds of modes, those excited along the orbifold only, along Σ only or along both (Flachi *et al.*, 2003). The masses m_Σ of the first graviton KK modes along Σ are of order $1/R$. However, the modes along the orbifold only have masses given by Morb $\sim ak$ (the curvature scale times the redshift factor). Thus the mass of these modes is of order $m_{\mathrm{orb}} \sim a$ Tev. Such small values do not conflict with observations because the coupling of these modes to matter is very suppressed (Flachi *et al.*, 2003).

In summary, the stabilization of the hierarchy works as follows in this scenario. We set the cutoff M and the SUSY breaking scale $\eta_{\mathrm{SUSY}} \lesssim M$ such that the curvature scale of the bulk is $k \sim$ TeV. Now we only need to find a mechanism that fixes

R_- small ($\sim 1/M$) and R_+ large ($R \sim 1/k$) (in Section 4 we discuss whether quantum effects can accomplish this). As a consequence, the masses of the gravition KK modes along Σ are $m_\Sigma \sim$ TeV, and for the modes along the orbifold are $m_{\rm orb} \sim a$ TeV, which is consistent with observations (Flachi *et al.*, 2003).

4. EFFECTIVE POTENTIAL AND STABILIZATION

The effective potential due to bulk fields propagating in the bulk can be computed in a variety of ways. Here we just display the result and defer the reader to Flachi *et al.* (2003) for details. The essential steps are the following. First, we find the KK spectrum for instance by means of the equation of motion. Then, we use dimensional regularization, and the techniques developed in the literature (Bordag *et al.*, 1996a,b; Elizalde *et al.*, 1993; Kirsten, 2001; Leseduarte and Romeo, 1994) to perform the sum over the KK modes. We obtain a divergent regularized potential that can be renormalized by subtracting a finite number of local operators. The outcome for the renormalized potential can be written as

$$V(R_\pm) = -\frac{1}{32\pi^2 R^4}\Bigg[\sum_{j=-1}^{\infty}\{(\beta_j - g_0 d_4 \delta_{4,j})[(kR_-)^j \ln(kR_-)^2 + (-kR_+)^j \ln(kR_+)^2] + (\gamma_j - \beta_j \ln(k/\mu)^2)[(kR_-)^j + (-kR_+)^j]\} + g_0 (kR)^4\{c_1 + a^4 c_2 - 2a^4 \mathcal{V}(a)\} + 2\sum_{l=1}^{\infty} g_l \hat{\lambda}_l^4 \mathcal{V}_l(a, R_-)\Bigg], \tag{16}$$

where

$$\mathcal{V}_l(a, R_-) = \int_1^\infty dz\, z(z^2-1)\ln\left(1 - \frac{k_\nu(\lambda_l z/ka)\, i_\nu(\lambda_l z/k)}{k_\nu(\lambda_l z/k)\, i_\nu(\lambda_l z/ka)}\right), \tag{17}$$

and λ_l are the eigenvalues (with degeneracy g_l) of the Laplacian $P_\Sigma \equiv (1/R^2)[-\Delta_\gamma + \xi \mathcal{R}_\gamma]$ appearing in the equations of motion (Flachi *et al.*, 2003). The coefficients β_j are defined as

$$\beta_j = \begin{cases} (1/2\sqrt{\pi})C_{5/2+D_2/2} & \text{for} \quad j=-1 \\ (3/2)\, C_{2+D_2/2} & \text{for} \quad j = 1 \\ -(d_j/\Gamma(j/2))C_{2-j/2+D_2/2} & \text{otherwise,} \end{cases} \tag{18}$$

where we understand that the Seeley–DeWitt coefficients $C_i(P_\Sigma)$ are zero if $i < 0$. The coefficients d_j can be obtained from the asymptotic expansion of the Bessel functions (see Flachi *et al.*, 2003, and c_1, c_2, and γ_j are constants and irrelevant for the present discussion (Flachi *et al.*, 2003).

We note that the computation can be carried out with no need to specify the internal space Σ. The dependence on Σ enters through the Seeley–DeWitt coefficients $C_i(P_\Sigma)$, and this is a consequence of having used the Mittag-Leffler expansion for the zeta function associated with P_Σ (Flachi *et al.*, 2003).

The result (16) is a potential $V(R_\pm)$ for the two moduli describing the background. Using $r_\pm \equiv kR_\pm$, it can be cast as

$$V(r_\pm) = -\frac{1}{32\pi^2 R^4}[V_+(r_+) + V_-(r_-) + v(r_+, r_-)], \tag{19}$$

where $v(R, r)$ contains the "nonlocal" part, and

$$V_\pm(r_\pm) = \sum_{j=-1}^{\infty} (\mp 1)^j \{\gamma_j r_\pm^j + (\beta_j - g_0 d_4 \delta_{j,4}) r_\pm^j \ln r_\pm^2 - \alpha_j^\pm r_\pm^j\}. \tag{20}$$

Here, the coefficients $\alpha_j^\pm$ are understood to be finite renormalization constants, and are nonzero when the corresponding logarithmic term is nonzero. This is dictated by β_j being zero or not (i.e., whether or not such a term is divergent), with the sole exception of $j = 4$. If $d_4 = 0$ and the Laplacian P_Σ has one zero eigenvalue, $g_0 = 1$, the logarithmic terms corresponding to $j = 4$ are not associated to any divergence of the effective action, and $\alpha_4^\pm = 0$. This situation arises, for example, when Σ is a torus.

Note that the sum goes from -1 to ∞ and we recall that from Eq. (18), all the β_j with $j > 4 + D_2$ vanish identically. Thus, the term $\beta_j r_-^j$ appears with j running from -1 to $D_2 + 4$, and the same holds for the terms with $\alpha_j^\pm$ (there are a finite number of divergent terms).

One interesting feature of the effective potential (19) in both cases with D_2 even and odd is that the two leading terms in the small $r_\pm$ limit (corresponding to $j = -1, 0$) do not depend on the mass m nor on the nonminimal coupling constant ξ. In the scenario that we are considering, with (broken) supersymmetry, number of the fermionic and bosonic degrees of freedom is the same. In this case, the terms with $j = -1$, 0, cancel identically. Thus. the sum in Eq. (20) actually begins at $j = 1$ rather than at $j = 1$.

As mentioned above, the effective potential contains a finite number of renormalization parameters $\alpha_j^\pm$. Their values are not computable from our effective theory. Rather, we shall fix them by requiring some renormalization conditions. which determine the values for the moduli as well. Since the moduli must be stabilized, we demand

$$\partial_{r+} V(r_\pm) = \partial_{r-} V(r_\pm) = 0, \tag{21}$$

and, to match the observed value of the effective four-dimensional cosmological constant, we shall impose

$$V(r_\pm)|_{\min} \simeq 10^{-122} m_P^4. \tag{22}$$

We are interested in the limit when the size of Σ is everywhere smaller than the orbifold size, $r_+ \lesssim 1$ and $r_- \ll 1$. One can show (Flachi *et al.*, 2003) that in this limit the nonlocal term $v(r_\pm)$ is exponentially suppressed, and we can approximate the potential by the "local" terms $V_\pm(r_\pm)$. Moreovcer, since we consider only the positive poweres of $r_\pm$ in $V_\pm$, the potential at the minimum is dominated by r_+. Then, conditions (21) and (22) reduce to

$$V'_+(r_+) = V'_-(r_-) = 0, \quad \text{and} \quad R^{-4}V_+(r_+)|_{\min} \simeq 10^{-122} m_P^4. \tag{23}$$

To investigate whether this potential can stbilize the moduli, we consider separately the casers with flat and curved Σ.

4.1. Flat Σ

This case corresponds to a toroidal compactification of a $4 + D_2 + 1$-dimensional RS model (with two codimension one brames). In this case, all the divergences have the same form, because all geometric invariants are constant and thus proportional to the brane tensions. Thus, there will appear a logarithmic term in the $(4 + D_2)$-th power of $r_\pm$. As can also be derived from Eqs. (16) and (18), setting $C_j = 0$ for all $j \neq 0$ and $g_0 = 1$, there is another logarithmic term corresponding to $j = 4$.

Thus, the expression for the potential reduces to

$$\begin{aligned} V_\pm(r_\pm) \approx \{ &\mp\gamma_1 r_\pm + \gamma_2 r_\pm^2 \mp \gamma_3 r_\pm^3 + (\gamma_4 - d_4 \ln r_\pm^2) r_\pm^4 + \cdots \\ &+ (\mp 1)^{4+D_2} \beta_{4+D_2} r_\pm^{4+D_2} \ln r_\pm^2 - \alpha^\pm_{4+D_2} r_\pm^{4+D_2} + \cdots \}. \end{aligned} \tag{24}$$

As we mentioned above, the renormalization constants $\alpha^\pm_{4+D_2}$ arise from a finite renormalization $\delta\tau_\pm$ of the brane tensions,

$$\delta\tau_\pm \int d^{4+D_2}x \sqrt{g_{(4+D_2)\pm}} = \frac{2\pi}{R^4} \int d^4x \delta\tau_\pm R_\pm^5,$$

so that $\alpha^\pm_{4+D_2} = (2\pi)^{D_2} \delta\tau_\pm / k^{4+D_2}$. The size of $\delta\tau_\pm$ is expected to be set by the SUSY breaking scale $\eta_{\rm SUSY}$ so that $\alpha^\pm_{4+D_2}$ are large in principle. Then, the main contributions to this potential arise from the fifth and the first powers. The extremum condition for the r_- modulus can be well approximated by

$$\delta\tau_- \sim \frac{1}{r_-^{D-2}} k^{D-1},$$

and assuming a natural value for $\delta\tau_-$ given by $\eta_{\rm SUSY}^{D-1}$ we obtain $\delta\tau_- \sim (k/M)(M/\eta_{\rm SUSY})^{1/2}$. Thus, for any dimension D the modulus R_- is stabilized without fine tuning near $1/M$. As for the modulus R_+, we expect the potential (24) to stabilize it near k once the fine tuning of $\delta\tau_+ \sim k^{D-1}$ needed for the cosmological constant is performed (Flachi *et al.*, 2003).

We can as well compute the masses for the moduli for an arbitrary number of flat internal dimensions. We find that the mass for ψ is always millimetric whereas

$m_\varphi \sim a$ TeV increases with D_2, ranging from 10 KeV for $D_2 = 1$ to 100 MeV for $D_2 = 6$. The coupling of φ to matter, of strength (see Eq. (11))

$$1/\left(h^{-1/(D-2)}\text{TeV}\right),$$

is comprised between $\sim 1/(10^4$ TeV) for $D_2 = 1$ and $\sim 1/(100$ TeV) for large D_2. This guarantees that it has not been produced at colliders or has any effect in star cooling.

4.2. Curved Σ

When Σ is not flat, besides the divergences proportional to brane tensions terms (giving rise to the power $r_\pm^{D-1}$ in $V_\pm$), the potential has more divergences. For instance, there can appear divergences proportional to curvature terms, which give rise to the powers $r_\pm^{D-3}$. Accordingly, terms with fewer powers of $r_\pm$ are due to higher powers of the curvature. and in general the effective potential takes the form

$$V_\pm(r_\pm) = \sum_{j=1}^{\infty}(\mp 1)^j\{\gamma_j r_\pm^j + (\beta_j r_\pm^j - g_0 d_4 \delta_{j,4}) \ln r_\pm^2\} - \sum_{j=1}^{D-1} \alpha_j^\pm r_\pm^j. \tag{25}$$

As in the previous case for the brane tensions, the size of the renormalization constants in front of these operators is expected to be of order the cutoff scale M (or $\eta_{\rm SUSY}$). Finite renormalization terms of boundary operators behave as

$$M^j \int d^{D-1}x \sqrt{g_{(D-1)\pm}}\mathcal{R}^{(D-1-j)/2} = \frac{1}{R^4}\int d^4x\,(MR_\pm)^j = \frac{1}{R^4} d^4x\,\alpha_j^\pm r_\pm^j,$$

and we conclude that the dimensionless renormalization constants in (25) are large, $\alpha_j^\pm \sim (M/k)^j \gg 1$. Thus, these terms are a series in $MR_\pm > 1$ rather than in $kR_\pm < 1$, the dominant terms being with the highest powers, i.e., the brane tension and the curvature terms. As a first approximation, we can neglect the remaining terms, and minimum condition for R_- is reached naturally for $R_- \sim 1/M$, which is what we need.

However, we see that to obtain $R_+ \sim 1/k$, we need to tune the ratio of α_{D-1} and α_{D-3}. Besides, the tuning corresponding to the cosmological constant is still needed.

In principle, we could consider the case when the heat kernel coefficient $C_1(P_\Sigma)$ is zero, which can happen for some value of the nonminimal coupling ξ. We see from (18) that in this case there is no divergence in the potential corresponding to the curvature terms.[3] Then, assuming that the next nonzero coefficient is C_2, the two powers that dominate the potential are $(MR_\pm)^{D-1}$ and $(MR_\pm)^{D-5}$. However, to stabilize R_+ near $1/k$, again we have to do one fine tuning. We can say that in

[3] The same thing cannot happen for the brane tension terms, since the corresponding coefficient is $C_0(P_\Sigma) = 1$ always.

general, the presence of any other divergence, besides the brane tension, spoils the efficiency of the potential in stabilizing the moduli at well-separated scales.

We conclude that, for curved Σ the potential can naturally stabilize the moduli but without a large hierarchy. In the next section, we analyze these models from the compactified, five-dimensional perspective. This analysis shows why curved or flat internal manifolds Σ lead to so different behaviors, and sheds some light on what kind model with curved Σ could stabilize the hierarchy naturally with quantum effects.

5. DIMENSIONAL REDUCTION AND THE 5D SCALAR-TENSOR MODEL

In Section 3, we have argued that there exists a range of energies where the theory is effectively 5 dimensional. In this section we show the dimensional reduction procedure from $D = 5 + D_2$ dimensions down to 5 dimensions, which allows contact with the language of Garriga *et al.*, (2003). The reduction from the higher dimensional theory (2) to 5 dimensions is performed by the compactification on the internal manifold Σ. This amounts to keeping only the Σ-zero modes of the fields defined in D dimensions.

In this section we denote collectively the four-dimensional Minkowski coordinates x^μ and the orbifold x^5 by x^α. Fo simplicity, we shall consider only the breathing mode of Σ in the internal components of the metric. As well, we shall freeze the $\{\alpha, i\}$ components (the graviphotons) to zero. Thus, the ansatz for the metric that we shall adopt depends on the internal coordinates X^i only through the background geometry on Σ, and on x^α through the five-dimensional graviton $g^{(5)}_{\alpha\beta}$, and a dilaton σ,

$$ds^2 = g^{(5)}_{\alpha\beta}(x)\, dx^\alpha\, dx^\beta + R^2\, e^{2\sigma(x)}\gamma_{ij}\, dX^i\, dX^j. \tag{26}$$

We shall also freeze the sigma model scalars to their value in the background, $\phi^a = \phi^a(X^i)$.

The action (2) corresponding to this ansatz can be easily written in the five-dimensional Einstein frame defined by $g^{\rm E}_{\alpha\beta} = e^{2D_2\sigma/3} g^{(5)}_{\alpha\beta}$, as

$$S_5 = -M_5^3 \int d^5x \sqrt{g_{\rm E}} \left\{ \mathcal{R}_{\rm E} + \frac{1}{2}(\partial\phi)^2_E + \Lambda_5\, e^{c\phi} \right\} \tag{27}$$

$$- \int d^4x \sqrt{g_{\rm E}+}\tau_{5+} e^{c\phi/2} - \int d^4x \sqrt{g_{\rm E}-}\tau_{5-} e^{c\phi/2}, \tag{28}$$

where we have performed the integration over X, the 5-dimensional Planck mass is $M_5^3 = v_\Sigma R^{D_2} M^{D-2}$, and we defined $c^2 = (2/3)D_2/(D_2 + 3)$, $\Lambda_5 = M^{2-D}\Lambda$, and $\tau_{5\pm} = v_\Sigma R^{D_2}\tau_\pm$. The canonical dilaton is given by $\phi = -(2D_2/3c)\sigma$ and the metrics on the branes induced by $g^{\rm E}_{\alpha\beta}$ are $g^{\rm E\pm}_{\mu\nu}$.

The action (27) coincides with the five-dimensional scalar-tensor model considered in Garriga *et al.* (2003). It was found there that this model has a solution with a power-law warp factor of the form

$$ds_{\mathrm{E}}^2 = a_{\mathrm{E}}^2(z)\,(dz^2 + \eta_{\mu\nu}\,dx^\mu\,dx^\nu),$$
$$\phi_0(z) = -\sqrt{6\beta(\beta+1)}\,\ln(z/z_0) \quad \text{with} \quad a_{\mathrm{E}}(Z) = (z/z_0)^\beta, \tag{29}$$

with[4] $\beta = 2/(3c^2 - 2) = -(D_2 + 3)/3$.

The brane operators induced by quantum effects on this background are given by positive powers of the extrinsic curvature scale (see, e.g., Garriga *et al.*, 2003) $\mathcal{K}_{\mathrm{E}\pm} = \beta/z_\pm a_{\mathrm{E}\pm} = \beta z_\pm^{-(\beta+1)}$,

$$\int d^4x\sqrt{g_{\mathrm{E}\pm}}\,\mathcal{K}_{\mathrm{E}\pm}^n = \int d^4x \left(\frac{z_\pm}{z_0}\right)^{(4-n)\beta} \frac{1}{z_\pm^n} \propto \int d^4x\, r_\pm^{4+(4/3)D_2-(n/3)D_2}, \tag{30}$$

where $n = 1, 2, 3, \ldots$, we used that the conformal coordinate $z = e^{ky}$ is the same in 5 and in D dimensions, and $a_\pm = 1/kz_\pm$. On the other hand, we have seen in Section 4 that the operators generated by the effective potential due to bulk fields in the model (2) are of the form

$$\int d^4x\sqrt{g_\pm}\mathcal{R}_\pm^N \propto \int d^4x\, r_\pm^{4+D_2-2N}, \tag{31}$$

where $\mathcal{R}_\pm$ is the intrinsic curvature computed with the induced metrics on each brane, $g_{\mu\nu}^\pm$. Here $N = 0, 1, \ldots [D/2]$, and [] denotes the integer part. Now we can identify that these operators correspond to a number of powers of the extrinsic curvature operator (30) given by

$$n = \frac{6}{D_2}N + 1.$$

We note that all the induced operators can be cast as powers of the extrinsic curvature for $D_2 = 1, 2, 3$, and 6 only, having in the $D_2 = 6$ case a one-to-one correspondence. For any other value of D_2, there exist higher dimensional local operators that are not simply powers of $\mathcal{K}_{\mathrm{E}\pm}$, but of some power of e^ϕ in the five-dimensional effective theory (27).

As well, it was noted in Garriga *et al.* (2003) that the path integral measure of a bulk scalar field in the effective five-dimensional theory (27) quantized on the warped vacuum configuration (29) is ambiguously defined. The nontrivial profile of the scalar ϕ permits to define many different conformal frames, all of them equivalent at the classical level. However, the path integral measure can be defined covariantly with respect to any of them. It turns out that the terms proportional to $z_\pm^{-4}$ In $z_\pm$ in the potential depend on this choice. Several arguments

[4] In terms of the proper coordinate (in the 5-dimensional Einstein frame) $y_{\mathrm{E}} \propto z^{\beta+1}$, $a_{\mathrm{E}}(y_{\mathrm{E}}) = (y_{\mathrm{E}}/y_0)^q$ with $q = 2/3c^2 = (D_2 + 3)/D_2$.

can be given in favor of possible "preferred" frames. For instance, with a measure covariant with respect to the five-dimensional Einstein frame metric $g^E_{\alpha\beta}$, this term is present. But if one chooses covariance with respect to $g^{(5)}_{\alpha\beta}$, there is no such term.

In the model presented here, there is no ambiguity in the choice of the measure since in the D-dimensional theory there is no scalar with nontrivial profile along the orbifold. In the computation presented here, the choice of the measure shows up (see Garriga *et al.*, 2003) when we subtract the divergences covariant precisely with respect to the higher dimensional Einstein frame metric $g^{(D)}_{MN}$ (Flachi *et al.*, 2003; Garriga *et al.*, 2003) The 5D result of Garriga *et al.* (2003) would conclude that using this frame, there is no logarithmic term in the potential. However, when we take into account the contribution from the Σ KK modes as well, we see that there is a remaining contribution of this form (see Eqs. (16) and (18). Anyhow, it should be noted that these logarithmic terms do not play a very relevant role in stabilizing the moduli.

6. CONCLUSIONS

In this paper, we have investigated a class of warped brane models with topology $M4 \times \Sigma \times S^1/Z_2$ where Σ is a D_2-dimensional compact manifold. In these models, both the compact and noncompact directions have an exponential warp factor, and two branes of codimension one are placed at the orbifold fixed points. Such a background can be obtained as a solution of a theory with a bulk nonlinear sigma model. A hedgehog configuration for the sigma model supports the curvature of space when the internal space Σ is curved. As in the five-dimensional models discussed in Garriga *et al.* (2003), this model contains two moduli, corresponding to the size of Σ on each brane, $R_\pm$. This is because in the limit of small Σ, these models reduce to some of those considered in Garriga *et al.* (2003).

We proposed a scenario where SUSY is broken at a scale just below the fundamental cutoff M. This makes the curvature scale k of the background to be a few orders of magnitude below M (Flachi *et al.*, 2003). In the presence of a stabilization mechanism that fixes $R_+ \sim 1/k$ and $R_- \sim 1/M$, then a large hierarchy is generated by a combination of redshift (Randall and Sundrum, 1999) and a large volume effects (Antoniadis *et al.*, 1998; Arkani-Haned *et al.*, 1998).

We have computed the contribution to the one-loop effective action from generic bulk scalar fields at lowest order (i.e., the Casmir energy). We find that (Flachi *et al.*, 2003), generically, the potential induced by bulk fields can generate sizable masses for the moduli compatible with a large hierarchy with no need of fine tuning if Σ is flat. If it is curved, the effective potential can naturally stabilize the moduli but no hierarchy is obtained unless the parameters are fine-tuned.

In the model we have considered, the size of the internal space Σ is everywhere smaller than the size of the orbifold. Therefore, there is a range of energy scales where the model is effectively five dimensional (as happens with the Hořava–Witten model Horava and Witter, 1996a,b; Lukas *et al.*, 1999 From the five-dimensional point of view (Flachi *et al.*, 2003), the model contains a dilaton field in the bulk, which causes a power-law warp factor, $a(y) \alpha\, y^q$, where y is the proper distance along the extra dimension. The power q is related to the number of additional dimensions through $q = (D_2 + 3)/(D_2$ (Garriga *et al.*, 2003), which leads to $1 < q \leq 4$. The quantum effects in five-dimensional models with power-law warp factors were investigated in Garriga *et al.* (2003), where it was found that the counterterms on the branes can stabilize a large hierarchy as long as $q \gtrsim 10$. This is consistent with the results of the present paper, which correspond to relatively small q. In this case, the large hierarchy can only be stabilized naturally if the internal space is flat. This case is special because the only possible counterterms are renormalizations of the higher dimensional brane tensions. This suggests that a large hierarchy may be obtained by considering more general warped models, where a larger power q is obtained upon reduction to five dimensions (Flachi *et al.*, 2003). In such cases, the stabilization of a hierarchy without fine tuning is expected even if the internal manifold Σ.

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